As a result of an instructional program in mathematics like that described in this guide, by the end of Grade 12, all students should be expected to complete work like the sample below:

CIGARETTE SMOKING

Assume that the average American smoker smokes 20 cigarettes per day and starts smoking at age 20.

You see a sign posted in the fitness center. It reads: "EVERY CIGARETTE YOU SMOKE REDUCES YOUR LIFE BY APPROXIMATELY 7 MINUTES!"

Is this a reasonable assertion or not? Show all of your work and explain how you arrived at your conclusion?

Your work will be evaluated on how well you have:

- devised a plan based on the assumptions above and your own assumptions; and
- explained the mathematics you used to support your decision.

CIGARETTE SMOKING

We figured that the average age for a human being living in the United States was 75 years old. If a person started smoking at 20 years old and smoked 20 cigarettes per day , he would be smoking for 55 years. We then knew that this person would smoke 7,300 cigarettes per year and 401,500 cigarettes in his life. We figured this out by multiplying 20 by 365 because there are 365 days in a year and he smokes 20 cigarettes per day. Then we multiplied 7,300 by 55 because he smokes 7,300 cigarettes per year and will live 55 years. To find out the number of minutes taken off his life because of smoking , we multiplied the number of cigarettes he smoked in his life (401,500) by 7 (Fitness Center's claim to the amount of minutes taken of your life for each cigarette smoked) , and got 2,810,500 minutes. Then we divided that number by 60 to get the number of hours smoking would take off his life and got 46,841.67 hours. Then we divided that number by 24 to get the number of days smoking would take off his life , and we got 1,951.74 days. Then we divided that number by 365 to find the total number of years that smoking would take off his life, and we got 5 years and 4 months.

So based on our results , we conclude that this claim by the Fitness Center is reasonable , that every cigarette you smoke reduces your life by 7 minutes.

CONTENT STANDARD 3: Estimation and Approximation

Students will make estimates and approximations, and judge the reasonableness of results.

K-12 PERFORMANCE STANDARDS

Educational experiences in **Grades K-4** will assure that students:

- explore, construct and use a variety of estimation strategies;
- recognize when estimation is appropriate and understand the usefulness of an estimate as distinct from an exact answer;
- use estimation to determine the reasonableness of an answer;
- visually estimate length, area, volume and angle using various referents; and
- apply estimation when working with quantities, measures and problems.

Educational experiences in **Grades 5-8** will assure that students:

- develop, apply and explain a variety of estimation strategies in problem situations involving quantities and measures;
- use estimation to predict outcomes and determine reasonableness of results;
- recognize when estimation is appropriate and understand the usefulness of an estimate as distinct from an exact answer; and
- determine whether a given estimate is an overestimate or underestimate.

Educational experiences in **Grades 9-12** will assure that students:

- assess the reasonableness of answers to problems arrived at using pencil-and-paper techniques, mental math, formulas, calculators or computers;
- develop, use and apply a variety of estimation strategies in problem situations;
- make reasonable estimates of the values of formulas, functions and roots; and
- recognize the limitations of estimation and assess the amount of error resulting from estimation.

ILLUSTRATIVE TASKS AND EXPERIENCES

As part of ongoing mathematics instruction in Grades K-4, students should have instructional experiences like the following:

1. How Many Buttons?

Ask students to work in groups to estimate how many buttons they think are in the classroom on a given day. Follow up with questions like:

- How did you arrive at this answer?
- How would you organize the data if you were to collect it?
- Will the number of buttons in our class be the same as in the other classes in the school? Why or why not?
- How many buttons do you think there are in the whole school today? Explain.
- What is the highest number of buttons that could be in the school today?
- What is the least number of buttons that could be in the school today?
- How did you come up with these answers?
- How would your numbers change if it was field day?
- How would your numbers change if it was school portrait day?
- How would your numbers change from fall to winter to spring? Explain.

Adapted with permission from *Measuring Up*, National Academy Press, 1993.

2. Estimate Needed?

Tell students to imagine that they are reporters for the school newspaper. They will need to decide how to report the numbers they use in their articles. For example, some numbers can only be estimates because the exact number is impossible to find. Often estimates are good enough, and sometimes a reporter must use an exact number. For each of the following situations explain in writing whether you would need to find an exact answer or whether an estimate would be good enough. Explain why.

- If a new classmate arrives tomorrow, how tall do you think he or she might be?
- How many people could fit in the auditorium for the holiday concert?
- How much milk do the students and teachers buy from the lunchroom on a given day? Would this amount of milk fit in a pot? a bath tub? a kiddy pool?
- What is the area of the playground?

Extension: Ask students to create two situations that would require an exact answer and two for which an estimate is good enough.

3. Counting On Frank

Read the book *Counting On Frank,* by Rod Clement (Gareth Stevens, Inc., 1991), with your class. Ask students to listen, as you read, for words that "speak" of estimation. With your class, make a list of estimation words you hear (they will be words such as *pretty big, a lot, about, almost, slightly less* and *guess*). Now ask students to write their own page to add to *Counting On Frank.* To get students started, you might offer suggestions like the following:

- In my closet I have around...
- For breakfast I ate about...
- I guess that so many of such an object could fit in...

Share your story with your classmates and ask them to name the word that "speaks" estimation.

4. Estimate Or Exact Number?

Tell students that they are to make believe they are reporters for the school newspaper. They have been assigned to write a story about the Little League baseball game on Saturday morning and they must report on such information as:

- the area of the baseball diamond;
- the perimeter of the baseball diamond;
- the area and perimeter of the entire field;
- the number of people at the game;
- the number of home runs for each team;
- the number of hot dogs sold during the game;
- the number of strikeouts pitched by each team;
- the amount of money taken in by the concession stand;
- the number of balloons sold by a local street vendor;
- the number of people it would take to stand around the edge of the baseball diamond; baseball field;
- the number of gallons of coffee consumed; and
- the number of cans of soda sold.

For each piece of information, record whether you need to have an actual number or if an estimate would be good enough? Tell why you made each decision.

As part of ongoing mathematics instruction in Grades 5-8, students should have instructional experiences like the following:

1. The Very Full Grocery Bag

Ask each student to bring to class a can or box of a grocery item (empty or full). This activity could be conducted as part of a food drive. Use the boxes and cans for the following activities.

- Order the boxes and cans by size from least volume to greatest volume using only spatial sense and estimation.
- Use volume formulas for rectangular prisms and cylinders to compute the volume of each box and can.
- Compare the estimated order with the actual order and discuss any discrepancies.
- Order the items by price from least expensive to most expensive after first estimating the order by price.
- Use the nutritional information on the labels to compute the cost per serving for each item. Group the items into high, medium and low cost per serving and compare and contrast the groupings.
- Estimate and then compute the cost per gram of protein for each of the items.
- Compare the nutritional value of your product with that of a classmate.
- Use the information on the label to develop a sales pitch for your product, including reasons why someone should buy it.

2. Planning A Trip

Ask students to select a destination at least 500 miles away that they would like to visit on a vacation. Distribute, or ask students to gather from a local travel agent or chamber of commerce, appropriate travel and lodging brochures for their destination. Use the maps and brochures to complete the following activities.

- Choose a hotel. What does the price of the hotel room include? Where is it located in reference to places you would want to go? Why are some hotels cheaper than others? What taxes are assessed?
- How would you travel to your destination? Using the scale of miles on a map, figure out approximately how far away your destination is. How long will it take you to get there? What is the least expensive mode of transportation? Don't forget to include food, gas and overnight stays on the way to your destination.
- Discuss the places you will want to visit while on your vacation and what the costs will be. How much will it cost to eat for one week? How will you get around? What other variables will you consider?
- You are now ready to plan a trip with your group! Budget all of your expenses for a one- or two-week stay at your destination. You may be given a budget. Make a chart that represents your trip expenses.
- Present a description of your trip, including expenses, to the class. Which group can plan the most economical trip? The class will evaluate the results.

3. Population Of The United States

In groups, ask students to discuss what they think is the approximate population of the United States. Place group estimates on a class chart. Next, inform students that the population of Connecticut is about 3.3 million, the population of New York is about 18 million and the population of Wyoming is about 460,000. Discuss whether or not students want to change their estimates.

Assign the following tasks for students to work on individually:

- You can ask for the population of five additional states. Which states would you choose? Explain your selections in writing.
- Use the information you have been given to determine a reasonable approximation of the U.S. population and explain what strategy you used to do so.
- Consider whether your estimation would be very different if you used only Eastern states? Western states? Large states? Small states? Explain.

Finally, in a group, ask students to compare their individual estimates with those of other group members.

As part of ongoing mathematics instruction in Grades 9-12, students should have instructional experiences like the following:

1. Owning A Car

Assign students the task of estimating the cost of buying and owning a used car for one year. Encourage students to use newspaper classified ads and sources of real data. Remind them to consider items such as sales tax, interest on a loan, gas, insurance, repairs, maintenance, etc. An extension might be to speculate about a job and the amount of income per hour needed in order to be able to afford the car.

2. The Dixie Cup Frustum

Inform students that the formula for the volume of a frustum of a cone – a shape like a Dixie cup – is $V = 1/3\ h\ (A + B + \sqrt{AB})$, where V is the volume, h is the height, and A and B are the areas of the top and bottom of the frustum. Let students use rulers and the formula to estimate the volume of various sized cups. Let students check their estimates by converting cubic centimeters to milliliters.

3. Wall Of Waste Paper

Share with students the estimate that people in the U.S. throw away enough office and writing paper each year to build a wall of paper 8 1/2 inches wide and 7 feet high from Los Angeles to New York. Ask students to convince the class that this is or is not reasonable and why. Describe any assumptions you make to support your argument or assumptions that the claim-maker must have made.

4. Do You Have The Energy?

Ask students to estimate how much they think their family spends each month on electricity and gas. Then distribute the data below and estimate a typical monthly energy bill, assuming that it is summer and houses need to be cooled. Compare your estimate to an actual energy bill for a summer month.

Computer	2 cents/hour	**Light Bulb**	1 cent/hour
Stereo	2 cents/hour	**Vacuum Cleaner**	9 cents/hour
Television	6 cents/hour	**Stove**	
Dishwasher	45 cents/load	Electric	15 cents/hour
Microwave	15 cents/hour	Gas	4 cents/hour
Refrigerator	3 cents/hour	**Cooling**	
Clothes Dryer		Air Conditioning	55 cents/hour
Electric	53 cents/load	Fan	7 cents/hour
Gas	10 cents/load	**Water Heater**	
Clothes Washer	7 cents/load	Electric	4 cents/hour
		Gas	2 cents/hour

Ask students the following questions:

- What appliances do you and your family use?
- How many hours a day do you use each appliance?
- Which appliances in your house use energy 24 hours a day?
- If an appliance you use cannot be found in the data, how could you estimate its costs?
- How can you organize the information to estimate your family's monthly energy bill?
- How close is your estimate to the actual bill? How could the estimate be adjusted?
- What situations may cause your utility bills to rise or fall?

PROTOTYPE ASSESSMENTS AND SAMPLES OF STUDENT WORK

As a result of an instructional program in mathematics like that described in this guide, by the end of Grade 4, all students should be expected to complete work like the sample below:

THE BIRTHDAY PARTY

You are going to have a birthday party. You plan to play several games at your birthday party. Since you know you will get quite a few gifts, you decide to have small prizes for the winner of each game. Your mom helps you buy the prizes and a box for each prize at the store, but you have the job of wrapping each box. You want to make each prize look as pretty as possible. You plan to wrap each prize and then tie a ribbon around each box.

You don't have a ruler at home to help you with the job of measuring. Describe the estimates you will have to make in order to cut a piece of ribbon long enough to go around a box and have enough left over to tie a pretty bow.

Some of the boxes are cubes, some are rectangular prisms, and some are in the shape of a cylinder. Be sure to describe how you estimated ribbon for each different box.

To successfully complete this task you are expected to provide:

- the estimated measures you used to decide how much ribbon to use; and
- an estimation procedure for each type of box.

Your work will be evaluated on how well you have:

- devised an estimation strategy for each box; and
- communicated your procedures.

Birthday Party Gifts

Cara

For the cube I measured one side with my finger. I found out that it was two fingers long. There were four sides. Two fingers inulitiplied by four equals eight fingers. Eight plus eight equals sixteen fingers. I figured I would need the same amount of ribbon to make a bow. Sixteen fingers plus sixteen fingers equals thirty-two fingers of ribbon needed to go around twice and tie a bow.

For the rectangular prism I measured the longer side. It was as long as my hand. Ends of the rectangle and the short sides were each one finger long. I would need twice the amount of ribbon as the long side to make a bow. I will need four hands and six fingers of ribbon to tie a bow around this box.

For the cylinder I measured the long sides to be as long as my pencil. The top to be as long as half pencil. It will take half as many pencils to make a bow. I will need nine pencils of ribbon.

As a result of an instructional program in mathematics like that described in this guide, by the end of Grade 8, all students should be expected to complete work like the sample below:

CONNECTICUT UNDER ATTACK

It has just been reported that Connecticut is under attack. The state is being invaded from the north, south, east and west. The governor has ordered every Connecticut citizen – man, woman and child – to the borders of the state to guard us from attack.

The governor asserts that, "We have enough people to stand shoulder to shoulder all around the state, including our borders with New York, Massachusetts, Rhode Island and the Long Island Sound coastline."

You know that:

- there are about 3.2 million Connecticut citizens;
- Connecticut is about 120 miles long and about 60 miles wide, but that our coastline is rather jagged; and
- there are 5,280 feet in one mile.

Determine whether or not the governor's claim is reasonable. Show your work and explain your reasoning.

Then name a few states where this attack-prevention strategy might NOT work. Explain why you chose these states.

3.2 mill = 3,200,000 citizens

$120 \cdot 2 = 240$ miles
$60 \cdot 2 = 120$ miles
≈ 360 miles around CT } Perimeter

$5280 \cdot 360 = 1,900,800$ feet around CT

about 1 person per foot 1 per./ft.

3,200,000
- 1,900,800
1,299,200 people left over

The governor's claim is correct. There would also be about 1,299,200 people left over to line around the state for a 2nd row.

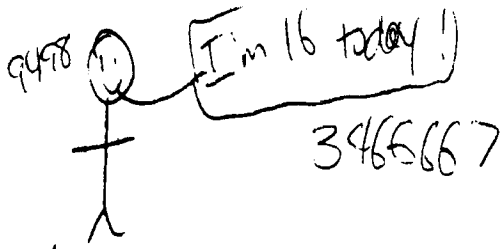
In some states like South + North Dakota, Minnesota, and definately Alaska, this plan would not work because the state's perimeter is greater than its population. That is, these states are much larger than CT, and have a much smaller population. In these states, you could probably drive a mack truck between any two people guarding the border.

As a result of an instructional program in mathematics like that described in this guide, by the end of Grade 12, all students should be expected to complete work like the sample below:

SWEET 16

You are designing birthday cards to sell in stores across the country. You think that a card celebrating 16th birthdays could be a popular item, but you aren't sure if there will be enough demand for the cards to make the project profitable.

You know that, according to the latest U.S. Census, there are approximately 260 million Americans alive today. What is a reasonable estimate for how many of these Americans are celebrating their 16th birthdays on a given day? Explain whether you should go ahead and produce the cards. Show your work and explain your reasoning.



260 million Americans
Avg life span is 75 260mil ÷ 75 = 3,466,667
365 days a year 9498 3,466,667 ÷ 365 = 9498 people

Since there are 260million living Americans today. ive assumed that the average life span is 75. Being that there are 260 mill. Americans alive, we divided that by 75 years old and came up with 3,466,667 people that are 16 years old this year. We then divided that number by the number of days in the year and came up with 9,498 people.

Since 9498 are celebrating their birthday we would assume that atleast 25% of those people (2375) will recieve one of our cards and we plan to sell our cards at $2.99 which would lead to a gross of $7,400.25/day, or $51,801.25/week, or $2,693,691/year you could assume that this is enough for production of the cards. One must also consider the extremely wide profit margin to be made on them (cost of card with labor would come out to be 1.50 per card)

5000/day

CONTENT STANDARD 4: Ratios, Proportions and Percents

Students will use ratios, proportions and percents to represent relationships between quantities and measures and solve problems involving ratios, proportions and percents.

K-12 PERFORMANCE STANDARDS

Educational experiences in **Grades K-4** will assure that students:	Educational experiences in **Grades 5-8** will assure that students:	Educational experiences in **Grades 9-12** will assure that students:
• describe simple ratios when comparing quantities.	• understand and use ratios, proportions and percents in a wide variety of situations; • develop, apply and explain methods for solving problems involving proportions and percents; • use and differentiate between fractional parts and ratios when comparing quantities; and • use dimensional analysis to identify and find equivalent rates.	• understand and explain the need for proportions and percents; • use ratios, proportions and percents to solve real-world problems; • use dimensional analysis and equivalent rates to solve problems; • describe direct and indirect variation and apply them to numerical, geometric and algebraic models and related problems; and • describe trigonometric ratios and apply them to measuring triangles.